Comment on "Conditions for Maximum Power Transfer"*

In recent correspondences by Shulman,[1] Castagnetto and Matheau,[2] there are discussions of the conditions for maximum power transfer. In Shulman's notation, the source impedance is $\boldsymbol{Z}_s = R_s + jX_s = R_s(1+jx_s)$, the load impedance is $\boldsymbol{Z} = R + jX = R_s(r+jx)$, P is the power delivered to the load, and P_0 is the maximum power available from the source. A very simple procedure is to plot $\boldsymbol{Z}' = r + j(x+x_s)$ on a Smith Chart. Maximum power transfer occurs when $\boldsymbol{Z}'$ is the closest to the center of the chart. In other words, maximum power transfer occurs when

$$\Gamma = \left| \frac{r + j(x + x_s) - 1}{r + j(x + x_s) + 1} \right|$$

is minimized, because

$$1 - \Gamma^2 = P/P_0.$$

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[1] Carl Shulman, "Conditions for maximum power transfer," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES (*Correspondence*), vol. MTT-9, pp. 453–454, September, 1961.

[2] L. Castagnetto, J. C. Matheau, and Carl Shulman, "Some remarks concerning 'Conditions for maximum power transfer,'" IEEE TRANS. ON MICROWAVE THEORY AND TECHNIQUES (*Correspondence*), vol. MTT-11, pp. 153–154; March, 1963.

Graphical Analysis of Q Circuits*

The parameters of a resonator which is coupled to a transmission line can be determined in a straightforward way from the measurement of the reflectance on the line.[1-5] However, graphical methods of evaluating the resonator parameters are based on the assumption that the inductance of the coupling loop can be neglected in comparison with the involved impedances. A simple method of graphical evaluation of the resonator parameters which takes into account the coupling inductance too will be described here.

The need for such a method arose in the course of the development of a wavemeter which had a resistance R_1 in series with the coupling loop. The input reflectance in the detuned short position is shown on Fig. 1. It is seen that the Q circle does not have its center on the real axis.

* Received May 20, 1963.

[1] L. R. Walker, "Output circuits," in "Microwave Magnetrons," G. B. Collins, Ed., McGraw-Hill Book Co., Inc., New York, N. Y., ch. 5, pp. 171–187; 1948.

[2] L. Malter and G. R. Brewer, "Microwave Q measurements in the presence of series lossses," *J. Appl. Phys.*, vol. 20, pp. 918–925; October, 1949.

[3] E. L. Ginzton, "Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y.; 1957. See especially, ch. 9.

[4] A. Singh, "An improved method for the determination of Q of cavity resonators," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 155–160; April, 1958.

[5] E. L. Ginzton, "Microwave Q measurements in the presence of coupling losses," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6 pp. 383–389 October, 1958.

Fig. 2 shows the equivalent circuit for the case under investigation. The position of the minimum voltage for the detuned resonator is marked with b—b, the detuned short position. The reflectances measured on the slotted line in the reference plane b—b produce the Q circle on the Smith Chart, denoted C_1 in Fig. 3. Its radius r_1 can be measured on the Γ scale which is usually added to the Smith Chart. The center of the Q circle is denoted S_1 while S_0 denotes the center of the Smith Chart.

The first step in the graphical analysis under consideration is to determine the circle C_2 which tangentially touches the unit circle (circumference of the Smith Chart) and the Q circle in the point F_∞. The easiest way of locating the circle C_2 is by simple trial. The straight line through points F_∞ and S_1 is drawn and the point S_2 is located, which makes $\overline{F_\infty S_2} = \overline{S_2 A}$. A is the touching point of the circle C_2 with the unit circle. The circle C_2 with the radius

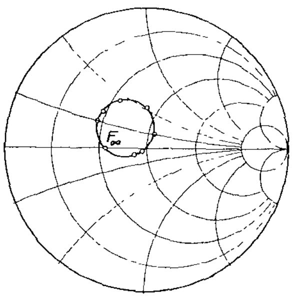

Fig. 1—Typical plot of the input reflectance of a resonator with coupling losses F_∞ is the reflectance of the detuned resonator.

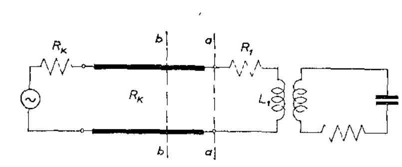

Fig. 2—Equivalent circuit of a resonator with coupling losses. a-a is the input terminals plane, b-b is the detuned short plane.

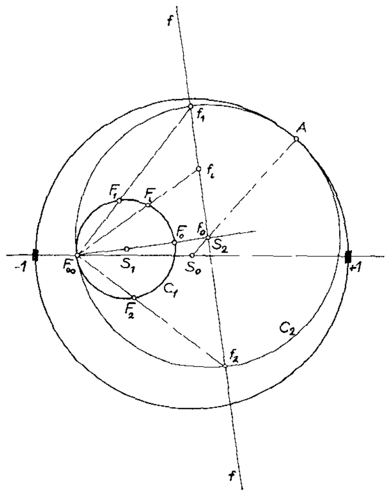

Fig. 3—Input reflectance in the detuned short plane (circle C_1). The radii of circles C_1 and C_2 determine the coupling coefficient. Also shown is the construction of a linear frequency scale.

r_2 is thus obtained. The series resistance R_1 in the coupling circuit is given by the relation

$$R_1 = R_K \left(\frac{1}{r_2} - 1 \right), \qquad (1)$$

where R_K denotes the characteristic resistance of the slotted line.

The coupling coefficient κ is here defined as the ratio of coupled resistance to cavity resistance. This corresponds to the situation described by Ginzton[3] in his Figure 9.3d. The coupling coefficient can be determined from the radii r_1 and r_2

$$\kappa = \frac{r_1}{r_2 - r_1}. \qquad (2)$$

Note that it is irrelevant whether the Q circle is overcoupled or undercoupled. All the relations given here are valid for both cases.

The straight line f-f drawn through S_2 perpendicularly to the line $\overline{F_\infty S}$, is provided with a linear frequency scale. The particular frequency f_t is determined by projecting on the scale f-f the point F_t out of the point F_∞. The frequency f_0, corresponding to F_0, is the resonant frequency of the loaded resonator.

By drawing two straight lines through the point F_∞ at 45° to the line $\overline{F_\infty S_2}$, one obtains the points F_1 and F_2. The corresponding frequencies f_1 and f_2 are then determined on the frequency scale f-f. The Q value of the loaded resonator follows from the familiar expression

$$Q_L = \frac{f_0}{f_2 - f_1}. \qquad (3)$$

For the sake of clarity, the rest of the analysis is explained on Fig. 4. From point A two straight lines are drawn at 45° to the line $\overline{AS_0}$, and a third line perpendicular to $\overline{AS_0}$. These three straight lines are denoted l_3, l_4 and l_5. The symmetry line between points F_∞ and A is drawn afterwards. The

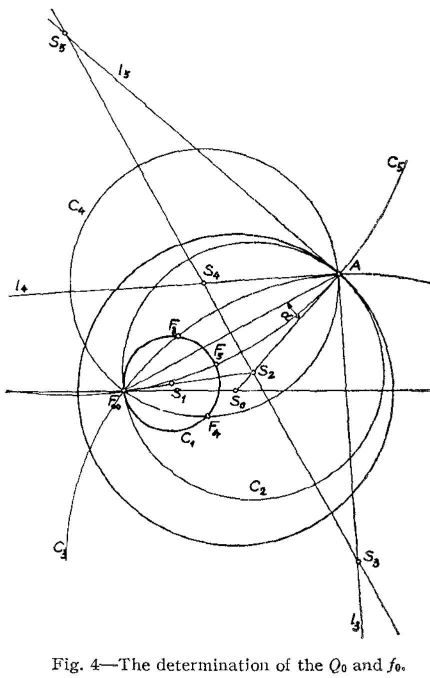

Fig. 4—The determination of the Q_0 and f_0.